Violação Espontânea de Simetria e Fases

Humberto Belich

PROFESSOR ASSOCIADO I - UFES

60 anos Helayel 2013

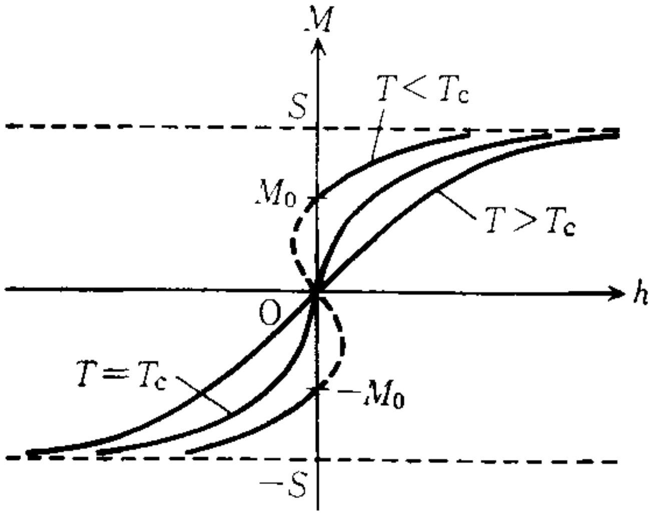
Orders in condensed matter systems

spontaneous symmetry breakdown

$$H = -J \sum_{(i,j)} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \boldsymbol{h} \cdot \sum_i \boldsymbol{S}_i$$

$$Z = \mathrm{tr}\, e^{-\beta H}, \text{ where } \beta = 1/T$$

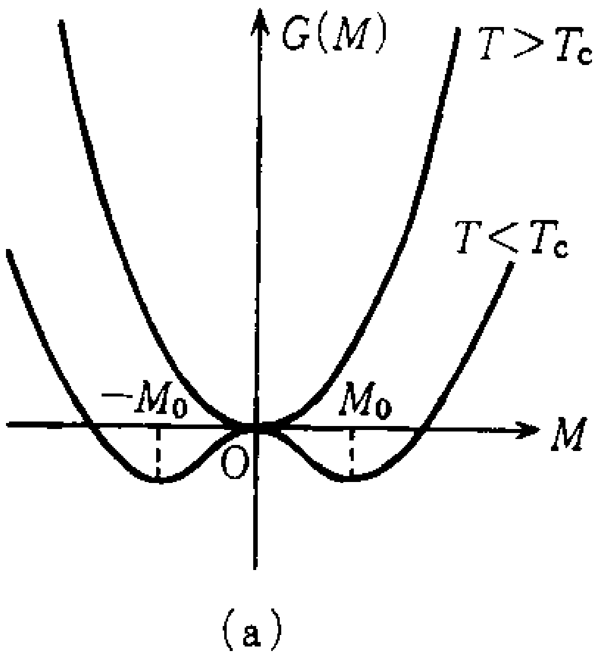
$$\mathrm{M} \equiv \frac{1}{N} \sum_i \langle \boldsymbol{S}_i \rangle = \frac{1}{N\beta} \frac{\partial F}{\partial \boldsymbol{h}}$$

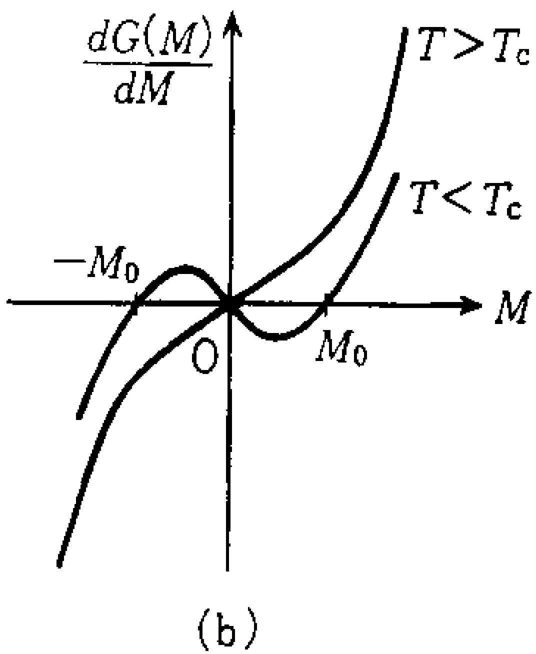


spontaneous magnetization

$$G(M) = aM^2 + bM^4 \qquad (b > 0)$$

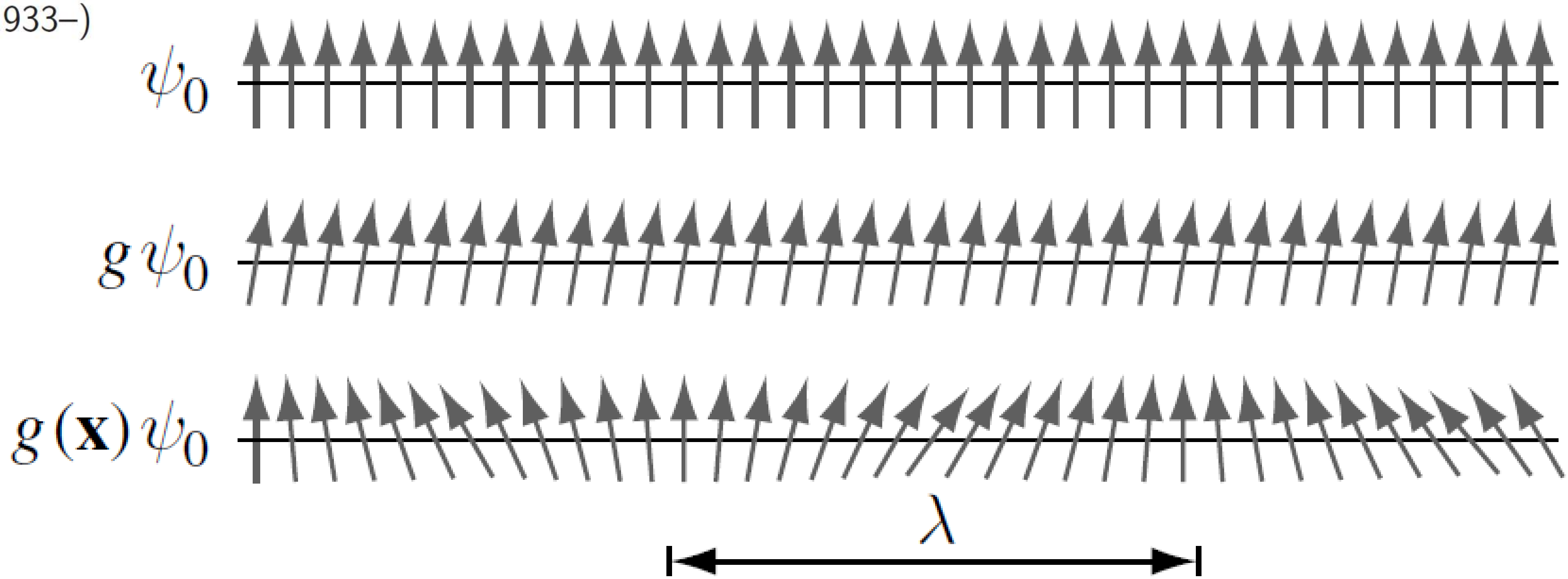
$$h = \frac{\partial G(M)}{\partial M}$$





Jeffrey R. Goldstone (1933–)

System	Broken symmetry	Goldstone mode
Magnet	Rotation	Spin wave
Solid	Translation	Phonon
Liquid helium	Global gauge invariance	Phonon
Superconductor	Local gauge invariance	N/A



System	*Broken symmetry*	*Goldstone excitation*
Crystal	Translational	Phonon
Ferromagnet	Rotational	Spin wave
Superfluid	Global gauge	Phonon
Superconductor	Local gauge	(Higgs mode)
Electro-weak[b]	Local gauge	(Higgs mode)
QCD[c]	Chiral	π mesons

Superfluid ^{4}He and superconductors

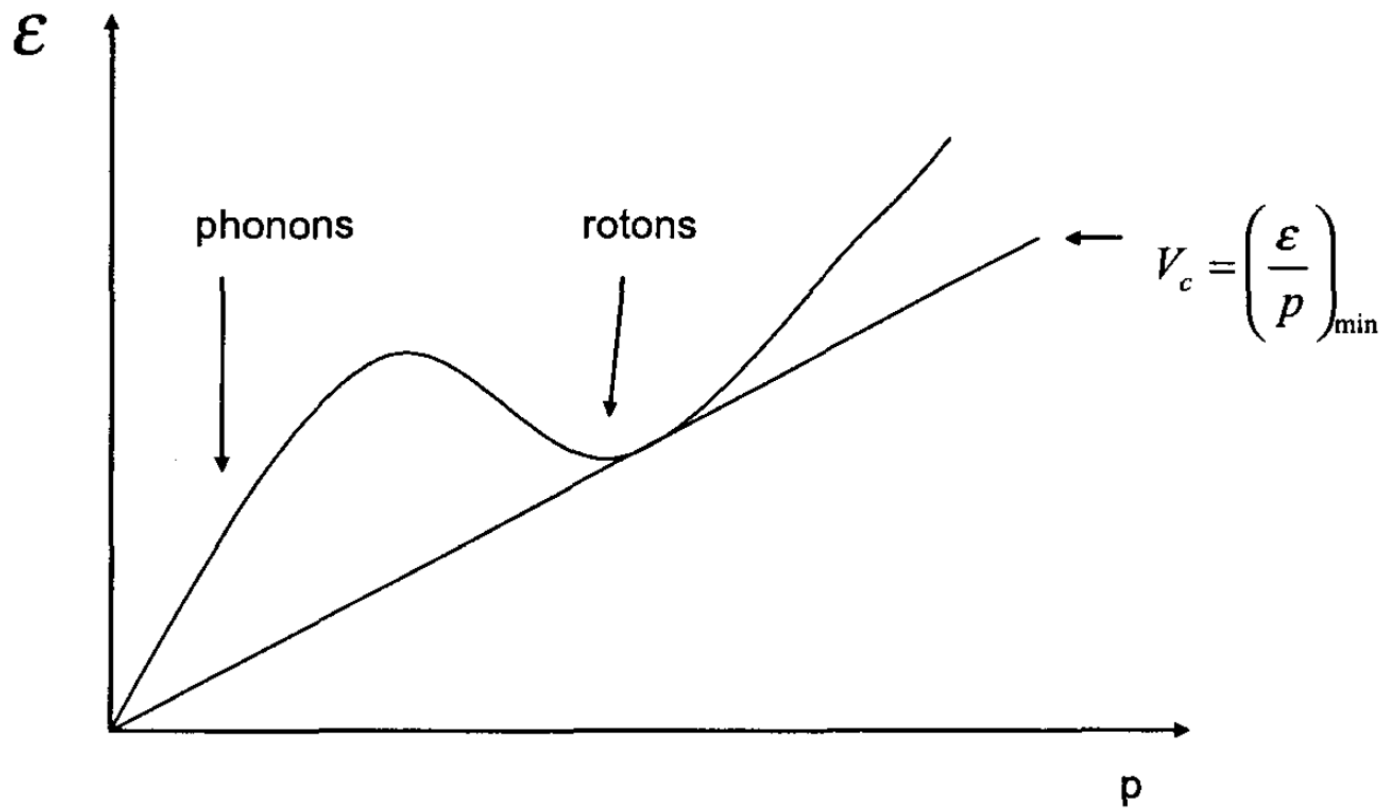
$$H = \int \mathrm{d}\boldsymbol{x}\, \phi^\dagger(\boldsymbol{x}) \left(-\frac{\nabla^2}{2m} - \mu \right) \phi(\boldsymbol{x}) + \frac{1}{2} \int \mathrm{d}\boldsymbol{x}\, \mathrm{d}\boldsymbol{y}\, \phi^\dagger(\boldsymbol{y}) \phi(\boldsymbol{y}) V(|\boldsymbol{x} - \boldsymbol{y}|) \phi^\dagger(\boldsymbol{x}) \phi(\boldsymbol{x})$$

$$\phi(\boldsymbol{x}) \to \mathrm{e}^{\mathrm{i}\chi} \phi(\boldsymbol{x})$$

Superfluidity

$$V_c = \left[\frac{\epsilon(p)}{p}\right]_{min}$$

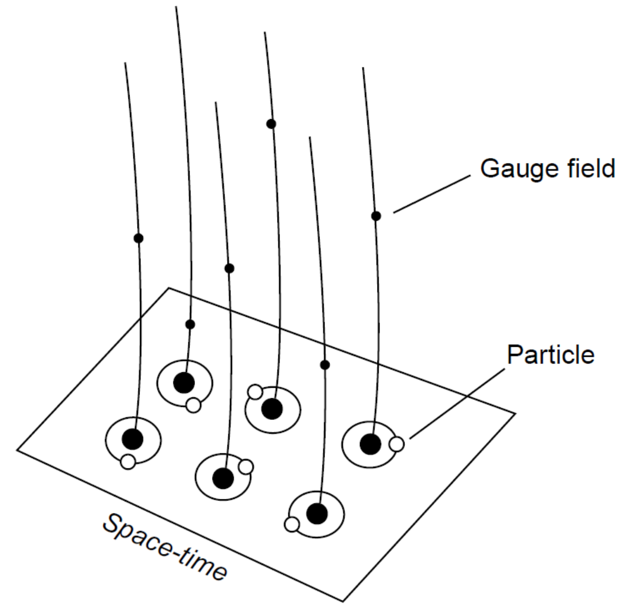
$$V_c = \frac{\Delta}{p_F}$$

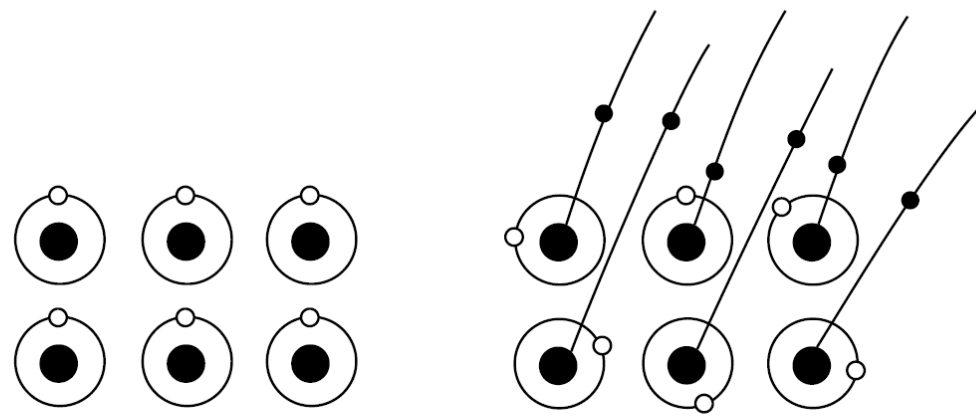


Global vs. local gauge invariance

$$A \to A + \partial\chi, \qquad \psi \to U\psi\,,$$

$$U = \exp\left(\frac{iq}{\hbar c}\chi\right) \qquad D = \partial + \frac{iq}{\hbar c}A$$

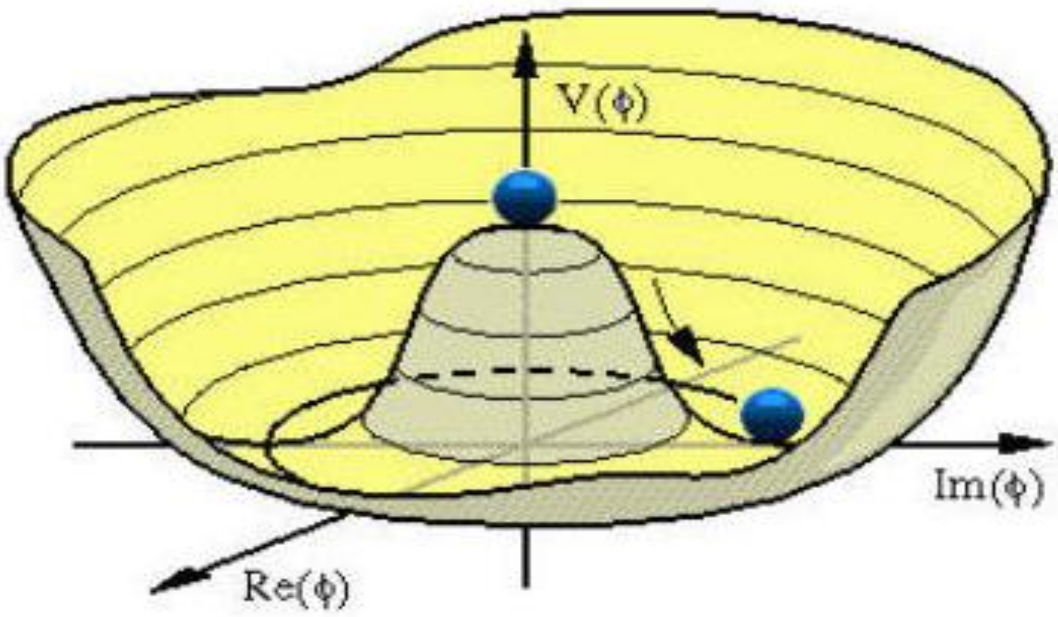


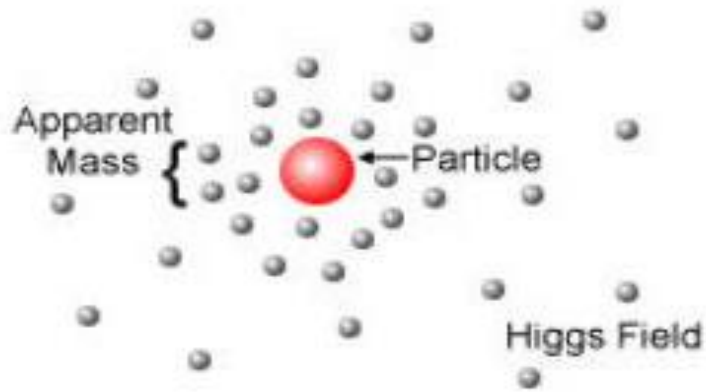


Global U(1) gauge transformation

Local U(1) gauge transformation

Superconductivity: the photon gets mass





Ginzburg-Landau Equation

$$F_s(\psi) = F_n + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{m^*}\left|\left(-i\hbar\nabla - \frac{2e\mathbf{A}}{c}\right)\psi\right|^2$$

$$a = \overline{a}(T - T_c)$$

$$b = \text{constant}$$

$$|\psi_0|^2 = -\frac{a}{b}$$

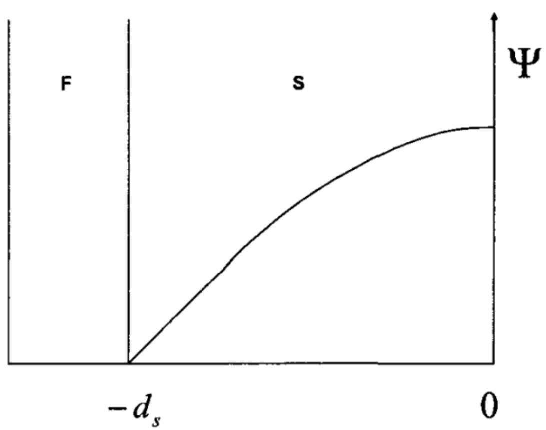
Vitaly L. Ginzburg

Boundary effect $$a\psi + b|\psi|^2\psi - \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = 0$$

Lev Davidovich Landau

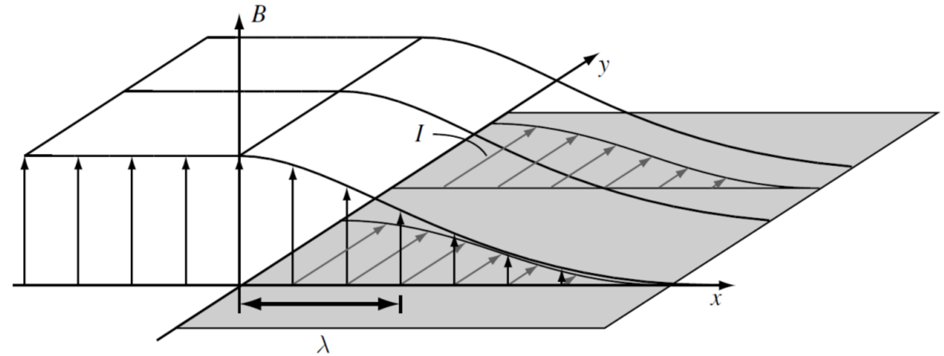
$$\psi = \psi \cos\left(\frac{x}{\xi(T)}\right)$$

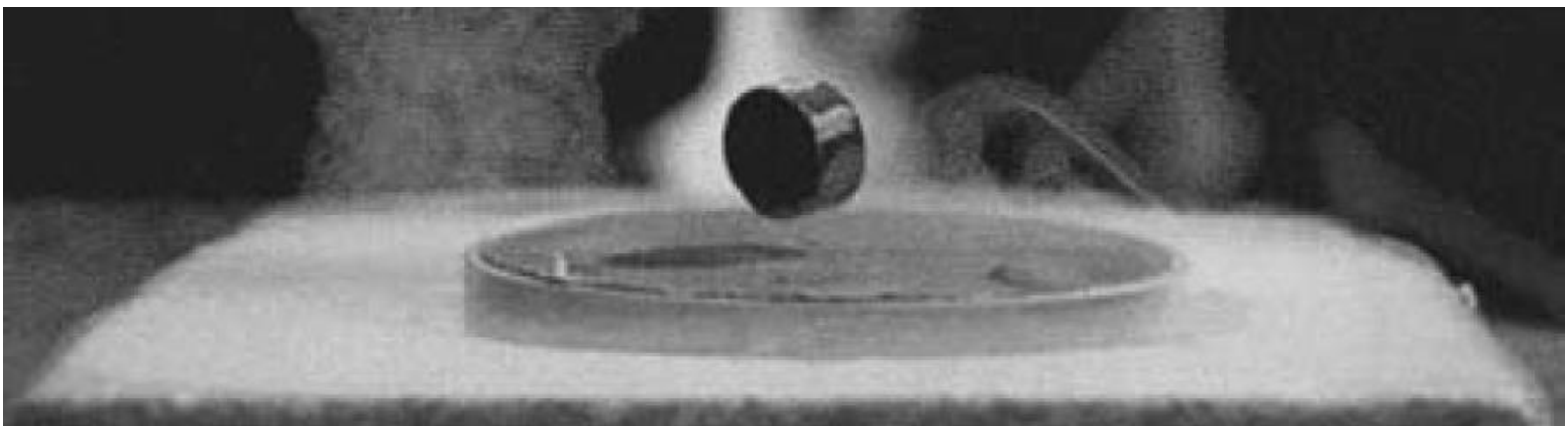
$$\xi^2(T) = \frac{\hbar^2}{2m|a|}$$

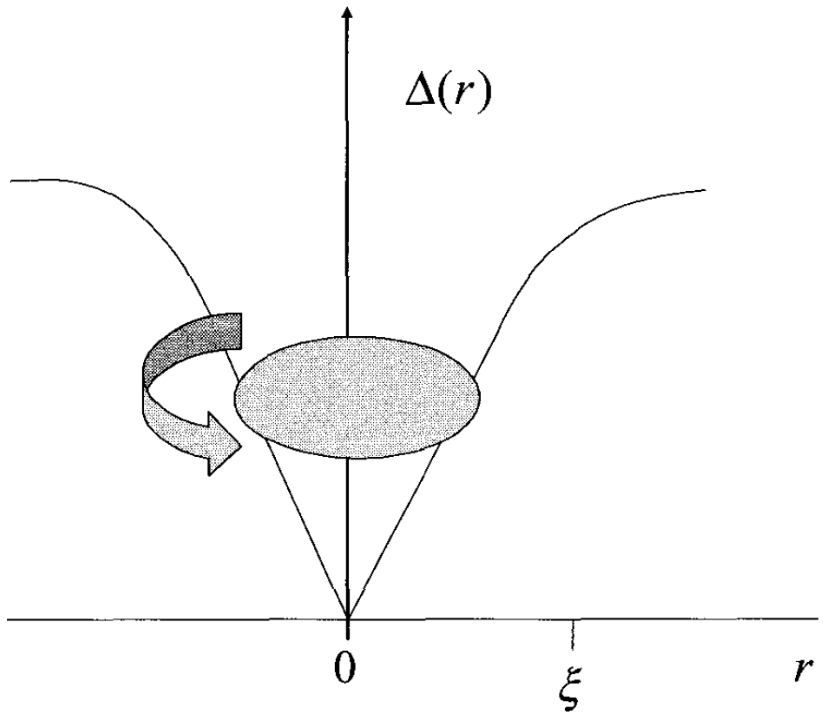


Superconductivity: the photon gets mass

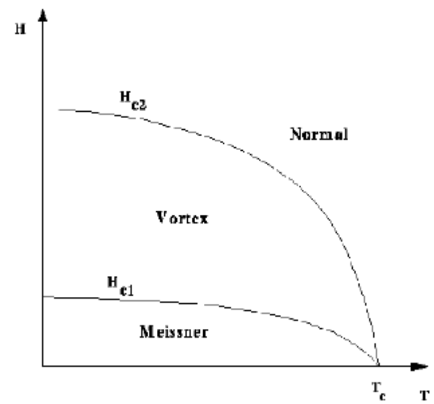
$$\nabla^2 \mathbf{A} + |\phi|^2 \mathbf{A} = 0$$

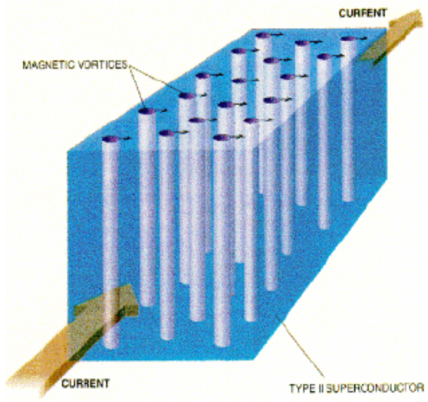






Type-II Superconductor





A current-carrying type II superconductor in the mixed state

When a current is applied to a type II superconductor (blue rectangular box) in the mixed state, the magnetic vortices (blue cylinders) feel a force (Lorentz force) that pushes the vortices at right angles to the current flow. This movement dissipates energy and produces resistance [from D. J. Bishop et al., Scientific American, 48 (Feb. 1993)].

http://phys.kent.edu/pages/cep.htm

BCS Theory of Superconductivity

The Nobel Prize in Physics 1972

- "for their jointly developed theory of superconductivity, usually called the BCS-theory"

John Bardeen	Leon Neil Cooper	John Robert Schrieffer
1/3 of the prize	1/3 of the prize	1/3 of the prize
USA	USA	USA
University of Illinois Urbana, IL, USA	Brown University Providence, RI, USA	University of Pennsylvania Philadelphia, PA, USA
b. 1908 d. 1991	b. 1930	b. 1931

- http://www.nobel.se/physics/laureates

ELECTRON-PHONON INTERACTIONS AND SUPERCONDUCTIVITY

Nobel Lecture, December 11, 1972

By JOHN BARDEEN

Departments of Physics and of Electrical Engineering

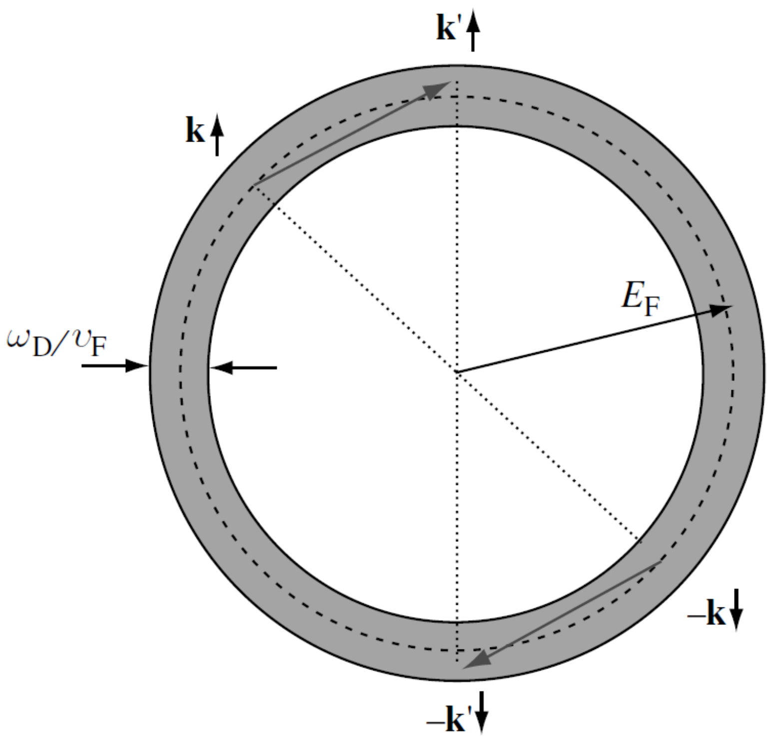
University of Illinois Urbana, Illinois

INTRODUCTION

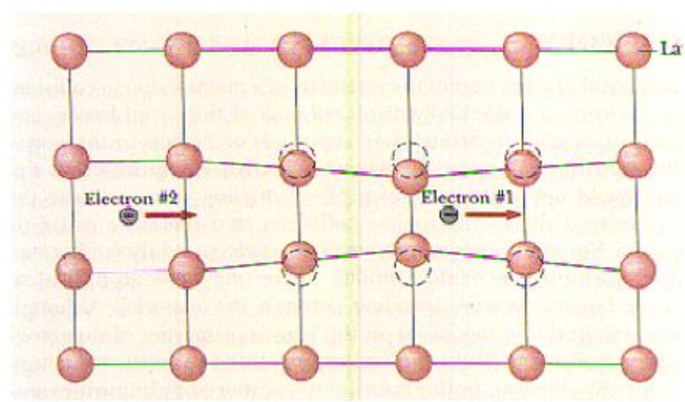
Our present understanding of superconductivity has arisen from a close interplay of theory and experiment. It would have been very difficult to have arrived at the theory by purely deductive reasoning from the basic equations of quantum mechanics. Even if someone had done so, no one would have believed that such remarkable properties would really occur in nature. But, as you well know, that is not the way it happened, a great deal had been learned about the experimental properties of superconductors and phenomenological equations had been given to describe many aspects before the microscopic theory was developed.

$$\hat{H} - \mu\hat{N} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \begin{pmatrix} \xi_{\mathbf{k}} & -\Delta \\ -\bar{\Delta} & -\xi_{\mathbf{k}} \end{pmatrix} \Psi_{\mathbf{k}} + \sum_{\mathbf{k}} \xi_{\mathbf{k}} + \frac{L^d |\Delta|^2}{g} \qquad \lambda_{\mathbf{k}} = (\Delta^2 + \xi_{\mathbf{k}}^2)^{1/2},$$

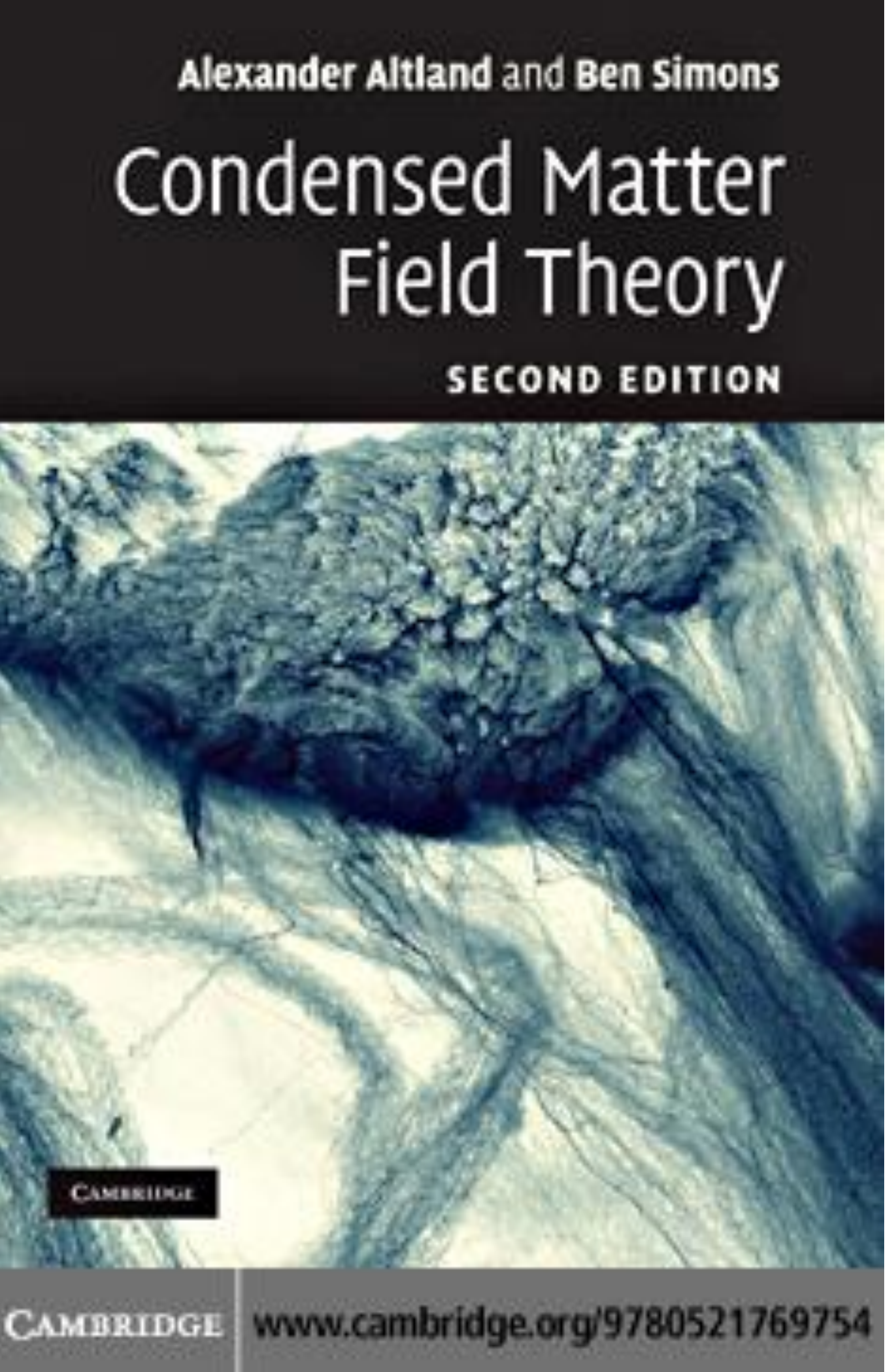
$$\hat{H} - \mu\hat{N} = \sum_{\mathbf{k}\sigma} \lambda_{\mathbf{k}} \alpha_{\mathbf{k}\sigma}^{\dagger} \alpha_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - \lambda_{\mathbf{k}}) + \frac{\Delta^2 L^d}{g}$$



The Electron-phonon Interaction



The origin of superconductivity in conventional superconductors



INTRODUCTION TO SUPERCONDUCTIVITY

Second Edition

Michael Tinkham

Rumford Professor of Physics

and

Gordon McKay Professor of Applied Physics

Harvard University

THE BCS GROUND STATE

$$|\psi_0\rangle = \sum_{k>k_F} g_{\mathbf{k}} c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow} |F\rangle$$

Since electrons obey Fermi statistics,

$$[c_{\mathbf{k}\sigma}, c^*_{\mathbf{k}'\sigma'}]_+ \equiv c_{\mathbf{k}\sigma} c^*_{\mathbf{k}'\sigma'} + c^*_{\mathbf{k}'\sigma'} c_{\mathbf{k}\sigma} = \delta_{\mathbf{k}\mathbf{k}'} \delta_{\sigma\sigma'}$$

$$[c_{\mathbf{k}\sigma}, c_{\mathbf{k}'\sigma'}]_+ = [c^*_{\mathbf{k}\sigma}, c^*_{\mathbf{k}'\sigma'}]_+ = 0$$

The particle number operator $n_{\mathbf{k}\sigma}$ is defined by

$$n_{\mathbf{k}\sigma} = c^*_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \tag{3.13}$$

the Cooper pairing built in is

$$|\psi_N\rangle = \sum g(\mathbf{k}_i, \ldots, \mathbf{k}_l) c^*_{\mathbf{k}_i\uparrow} c^*_{-\mathbf{k}_i\downarrow} \cdots c^*_{\mathbf{k}_l\uparrow} c^*_{-\mathbf{k}_l\downarrow} |\phi_0\rangle$$

BCS took as their form for the ground state

$$|\psi_G\rangle = \prod_{\mathbf{k}=\mathbf{k}_1,\ldots,\mathbf{k}_M} (u_{\mathbf{k}} + v_{\mathbf{k}} c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow}) |\phi_0\rangle$$

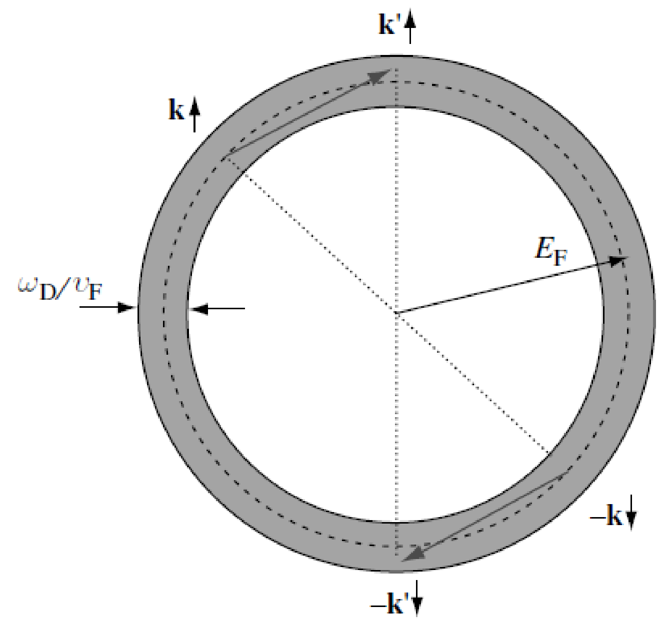
$$|\psi_\varphi\rangle = \prod_{\mathbf{k}} (|u_k| + |v_k| e^{i\varphi} c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow}) |\phi_0\rangle$$

$$c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} = b_{\mathbf{k}} + (c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - b_{\mathbf{k}}) \qquad b_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle_{\text{av}}$$

$$\mathscr{H}_M = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^*_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{kl}} V_{\mathbf{kl}} (c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow} b_{\mathbf{l}} + b^*_{\mathbf{k}} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow} - b^*_{\mathbf{k}} b_{\mathbf{l}})$$

Comprising two fermions, the electron–electron bound states, known as **Cooper pairs**, mimic the behavior of bosonic composite particles.[18] At low temperatures, these quasi-bosonic degrees of freedom form a condensate which is responsible for the remarkable properties of superconductors, such as perfect diamagnetism.

To appreciate the tendency to pair formation in the electron system, consider the diagram shown in the figure below. The region of attractive correlation is indicated as a shaded ring of width $\sim \omega_D/v_F$. Now, consider a two-electron state $|\mathbf{k}\uparrow, -\mathbf{k}\downarrow\rangle$ formed by two particles of (near) opposite momentum and opposite spin.[19] Momentum conserving scattering of the constituent particles may lead to the formation of a new state $|(\mathbf{k}+\mathbf{p})\uparrow, -(\mathbf{k}+\mathbf{p})\downarrow\rangle \equiv |\mathbf{k}'\uparrow, -\mathbf{k}'\downarrow\rangle$ of the same, opposite-momentum structure. Crucially, the momentum transfer $\mathbf{p}$ may trace out a large set of values of $\mathcal{O}(k_F^{d-1}\omega_D/v_F)$ without violating the condition that the final states be close to the Fermi momentum. (By contrast, if the initial state had not been formed by particles of opposite momentum, the phase space for scattering would have been greatly diminished.) Remembering our previous discussion of the RPA approximation, we recognize a familiar mechanism: an *a priori* weak interaction may amplify its effect by conspiring with a large phase space volume.



To explore this mechanism in quantitative terms, we will adopt a simplified model defined by the Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} \hat{n}_{\mathbf{k}\sigma} - \frac{g}{L^d} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^\dagger_{\mathbf{k}+\mathbf{q}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'+\mathbf{q}\downarrow} c_{\mathbf{k}'\uparrow}, \tag{6.14}$$

Mean-field theory of superconductivity

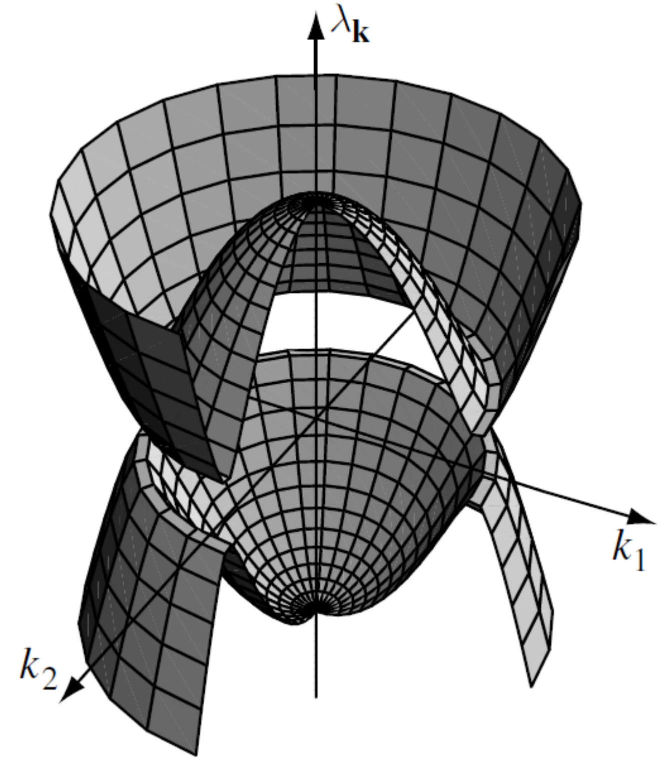
The discussion of the previous section suggests that, at the transition, the system develops an instability towards pair binding, or "condensation." In the next section we build on this observation to construct a quantitative approach, based on a Hubbard–Stratonovich decoupling in the Cooper channel. However, for the moment, let us stay on a more informal level and *assume* that the ground state $|\Omega_s\rangle$ of the theory is characterized by the presence of a macroscopic number of Cooper pairs. More specifically, let us assume that the operator $\sum_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}$ acquires a non-vanishing ground state expectation value,

$$\Delta = \frac{g}{L^d} \sum_{\mathbf{k}} \langle \Omega_s | c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} | \Omega_s \rangle, \quad \bar{\Delta} = \frac{g}{L^d} \sum_{\mathbf{k}} \langle \Omega_s | c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} | \Omega_s \rangle, \tag{6.17}$$

$$\hat{H} - \mu\hat{N} \simeq \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \left(\bar{\Delta} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} + \Delta c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right) \right] + \frac{L^d |\Delta|^2}{g},$$

known (in the Russian literature) as the **Bogoliubov** or **Gor'kov** Hamiltonian after its authors (while the terminology **Bogoliubov–de Gennes Hamiltonian** has become more widespread in the Anglo-Saxon literature, reflecting the promotion of the mean-field description by de Gennes).

Indeed, although perfectly Hermitian, the Gor'kov Hamiltonian does not conserve particle number. Instead, pairs of particles are born out of, and annihilated into, the vacuum. To bring the mean-field Hamiltonian to a diagonal form, we proceed in a manner analogous to that of Section 2.2 (where appeared a Hamiltonian of similar structure, namely $a^\dagger a + aa + a^\dagger a^\dagger$). Specifically, let us recast the fermion operators in a two-component **Nambu spinor** representation



$$\Psi^\dagger_{\mathbf{k}} = \left(c^\dagger_{\mathbf{k}\uparrow}, c_{-\mathbf{k}\downarrow} \right), \quad \Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix},$$

comprising ↑-creation and ↓-annihilation operators in a single object. It is then straightforward to show that the Hamiltonian assumes the bilinear form

$$\hat{H} - \mu \hat{N} = \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \begin{pmatrix} \xi_{\mathbf{k}} & -\Delta \\ -\bar{\Delta} & -\xi_{\mathbf{k}} \end{pmatrix} \Psi_{\mathbf{k}} + \sum_{\mathbf{k}} \xi_{\mathbf{k}} + \frac{L^d |\Delta|^2}{g}.$$

Now, being bilinear in the Nambu operators, the mean-field Hamiltonian can be brought to a diagonal form by employing the unitary transformation[22]

$$\chi_{\mathbf{k}} \equiv \begin{pmatrix} \alpha_{\mathbf{k}\uparrow} \\ \alpha_{-\mathbf{k}\downarrow}^{\dagger} \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathbf{k}} & \sin\theta_{\mathbf{k}} \\ \sin\theta_{\mathbf{k}} & -\cos\theta_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c_{-\mathbf{k}\downarrow}^{\dagger} \end{pmatrix} \equiv U_{\mathbf{k}} \Psi_{\mathbf{k}},$$

(under which the anti-commutation relations of the new electron operators $\alpha_{\mathbf{k}\sigma}$ are maintained – exercise). Note that the operators $\alpha_{\mathbf{k}\uparrow}^{\dagger}$ involve superpositions of $c_{\mathbf{k}\uparrow}^{\dagger}$ and $c_{-\mathbf{k}\downarrow}$, i.e. the quasi-particle states created by these operators contain linear combinations of particle and hole states. Choosing Δ to be real[23] and setting $\tan(2\theta_{\mathbf{k}}) = -\Delta/\xi_{\mathbf{k}}$, i.e. $\cos(2\theta_{\mathbf{k}}) = \xi_{\mathbf{k}}/\lambda_{\mathbf{k}}$, $\sin(2\theta_{\mathbf{k}}) = -\Delta/\lambda_{\mathbf{k}}$, where

$$\lambda_{\mathbf{k}} = (\Delta^2 + \xi_{\mathbf{k}}^2)^{1/2}, \tag{6.18}$$

the transformed Hamiltonian takes the form (exercise)

$$\hat{H} - \mu \hat{N} = \sum_{\mathbf{k}\sigma} \lambda_{\mathbf{k}} \alpha_{\mathbf{k}\sigma}^{\dagger} \alpha_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - \lambda_{\mathbf{k}}) + \frac{\Delta^2 L^d}{g}. \tag{6.19}$$

$$A \begin{pmatrix} \alpha_{k\uparrow} \\ \alpha^{\dagger}_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^{\dagger}_{-k\downarrow} \end{pmatrix}$$

$$\left(c^{\dagger}_{k\uparrow} \,,\, c_{-k\downarrow} \right) \begin{pmatrix} \xi_k & -\Delta \\ -\bar{\Delta} & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^{\dagger}_{-k\downarrow} \end{pmatrix} = \qquad M \to ($$

$$\Rightarrow \begin{pmatrix} c_{k\uparrow} \\ c^{\dagger}_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \alpha_{k\uparrow} \\ \alpha^{\dagger}_{-k} \end{pmatrix}$$

$$A' = M^{-1} A M$$

$$= \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \begin{pmatrix} \xi & -\Delta \\ -\bar{\Delta} & -\xi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \qquad M^{-1} \to \begin{pmatrix} u_k^{*} & v_k \\ -v_k^{*} & u_k \end{pmatrix}$$

$$\begin{pmatrix} \xi\cos - \Delta\sin & \xi\sin + \Delta\cos \\ -\bar{\Delta}\cos - \xi\sin & -\bar{\Delta}\sin + \xi\cos \end{pmatrix}$$

$$= \left| \begin{matrix} \xi\cos^2 - \Delta\sin\cos - \Delta\sin\cos - \xi\sin^2 \,; & \xi\sin\cos + \Delta\cos^2 - \bar{\Delta}\sin^2 + \xi\sin\cos \\ \xi\sin\cos - \Delta\sin^2 + \Delta\cos^2 + \xi\sin\cos \,, & \xi\sin^2 + \Delta\sin\cos + \Delta\sin\cos - \end{matrix} \right.$$

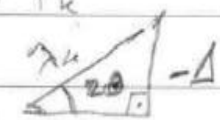
$$= \begin{pmatrix} \xi\overbrace{(\cos^2 - \sin^2)}^{\cos 2\theta = \xi/\lambda} - \overbrace{2\sin\cos}^{\sin 2\theta}\Delta \,, & 2\sin\cos\xi + \Delta\overbrace{(\cos^2 - \sin^2)}^{\cos 2\theta} \\ \overbrace{2\sin\cos}^{\sin 2\theta}\xi + \Delta\overbrace{(\cos^2 - \sin^2)}^{\cos 2\theta} \,, & -\xi\overbrace{(\cos^2 - \sin^2)}^{\cos 2\theta} + \overbrace{2\sin\cos}^{\sin 2\theta}\Delta \end{pmatrix}$$

Supõe: $\Delta = \bar{\Delta}$ e $\cos 2\theta = \dfrac{\xi}{\lambda}$, $\sin 2\theta = -\dfrac{\Delta}{\lambda}$ $\quad tg\, 2\theta = -\dfrac{\Delta}{\xi}$; $\lambda_k = \sqrt{\Delta_k^2 + \xi_k^2}$

$$= \begin{pmatrix} \xi\dfrac{\xi}{\lambda} - \Delta\left(-\dfrac{\Delta}{\lambda}\right) \,, & \left(-\dfrac{\Delta}{\lambda}\right)\xi + \Delta\,\xi/\lambda \\ \left(-\dfrac{\Delta}{\lambda}\right)\xi + \Delta\dfrac{\xi}{\lambda} \,, & -\xi\left(\dfrac{\xi}{\lambda}\right) + \Delta\left(-\dfrac{\Delta}{\lambda}\right) \end{pmatrix}$$

$$(\xi^2 + \Delta^2$$

Supõe: $\Delta = \bar{\Delta}$ e $\cos 2\theta = \frac{\xi}{\lambda}$, $\sin 2\theta = -\frac{\Delta}{\lambda}$, $\operatorname{tg} 2\theta = -\frac{\Delta}{\xi}$; $\lambda_k = \sqrt{\Delta_k^2 + \xi_k^2}$



$$= \begin{pmatrix} \xi \frac{\xi}{\lambda} - \Delta\left(-\frac{\Delta}{\lambda}\right) , & \left(-\frac{\Delta}{\lambda}\right)\xi + \Delta\, \xi/\lambda \\ \left(-\frac{\Delta}{\lambda}\right)\xi + \Delta \frac{\xi}{\lambda} , & -\xi\left(\frac{\xi}{\lambda}\right) + \Delta\left(-\frac{\Delta}{\lambda}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\xi^2}{\lambda} + \frac{\Delta^2}{\lambda} & 0 \\ 0 & -\left(\frac{\xi^2}{\lambda} + \frac{\Delta^2}{\lambda}\right) \end{pmatrix} = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda \end{pmatrix};$$

$$\begin{pmatrix} \alpha^+_{k\uparrow} & \alpha_{-k\downarrow} \end{pmatrix} \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda \end{pmatrix} \begin{pmatrix} \alpha_{k\uparrow} \\ \alpha^+_{-k\downarrow} \end{pmatrix}$$

$\alpha_{k\uparrow} = \cos\theta\, c_{k\uparrow} + \sin\theta\, c^+_{-k\downarrow}$

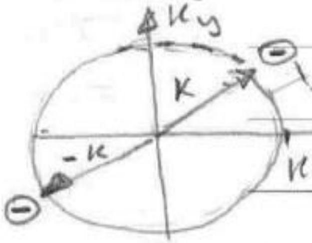
$\alpha_{-k\downarrow} = \sin\theta\, c^+_{k\uparrow} - \cos\theta\, c_{-k\downarrow}$

$$\begin{pmatrix} \lambda\, \alpha_{k\uparrow} \\ -\lambda\, \alpha^+_{-k\downarrow} \end{pmatrix} = \lambda\, \alpha^+_{k\uparrow} \alpha_{k\uparrow} - \lambda\, \alpha_{-k\downarrow} \alpha^+_{-k\downarrow}$$

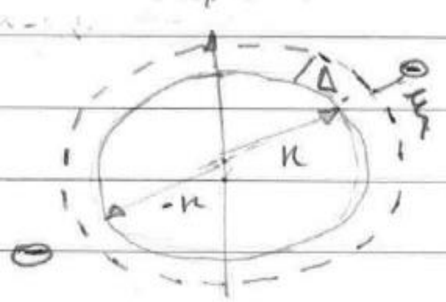
$$= \lambda_k\, \alpha^+_{k\uparrow} \alpha_{k\uparrow} + \lambda_{-k}\, \alpha^+_{-k\downarrow} \alpha_{-k\downarrow}$$

$$= \sum_{k\sigma} \lambda_k\, \alpha^+_{k\sigma} \alpha_{k\sigma}$$

Antes

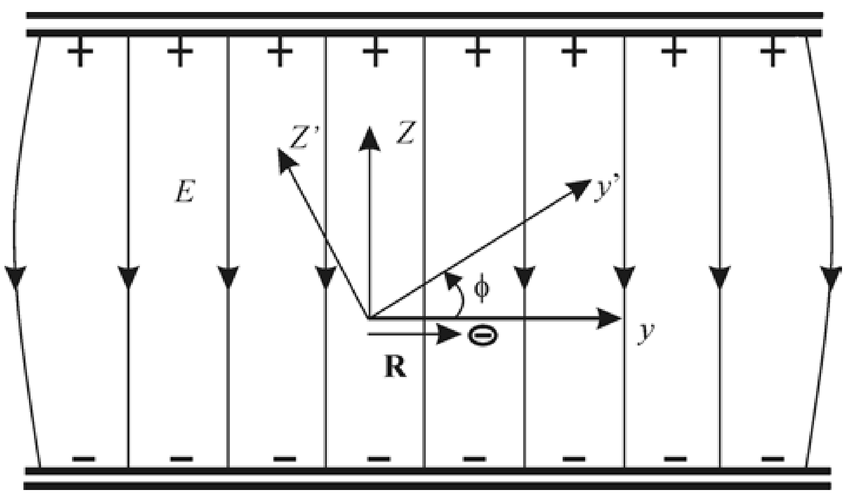


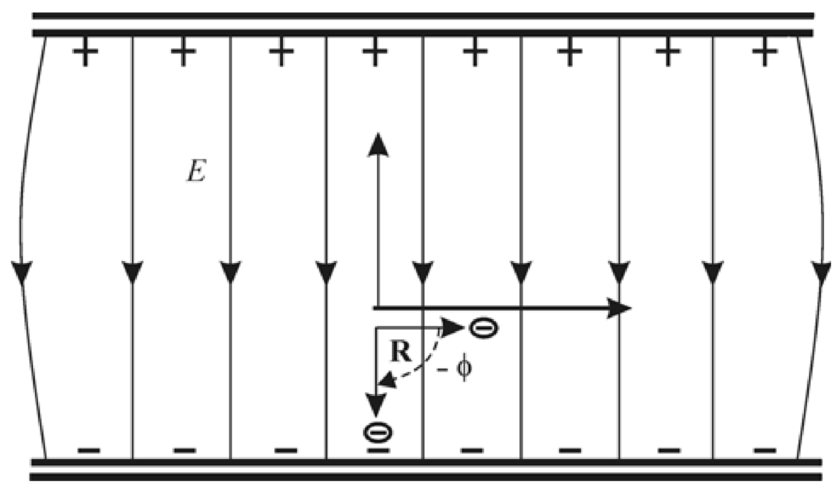
Depois



Violação da simetria de Lorentz

Revista Brasileira de Ensino de Física, v. 29, n. 1, p. 57-64, (2007)

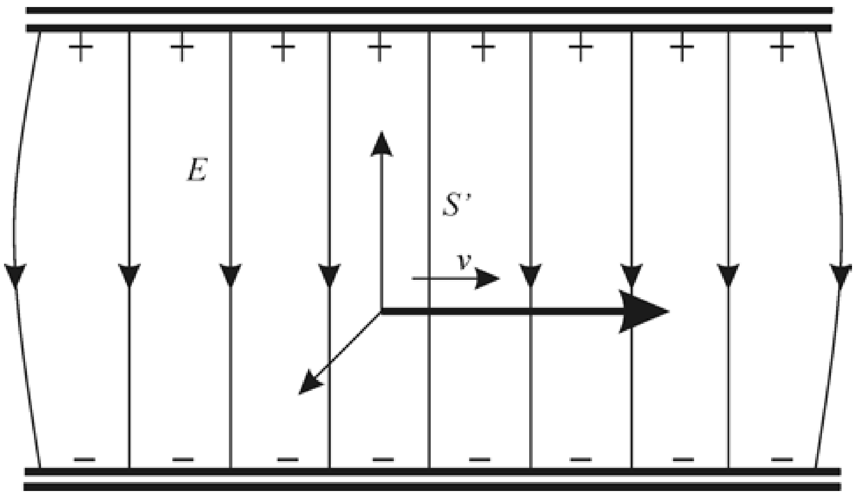


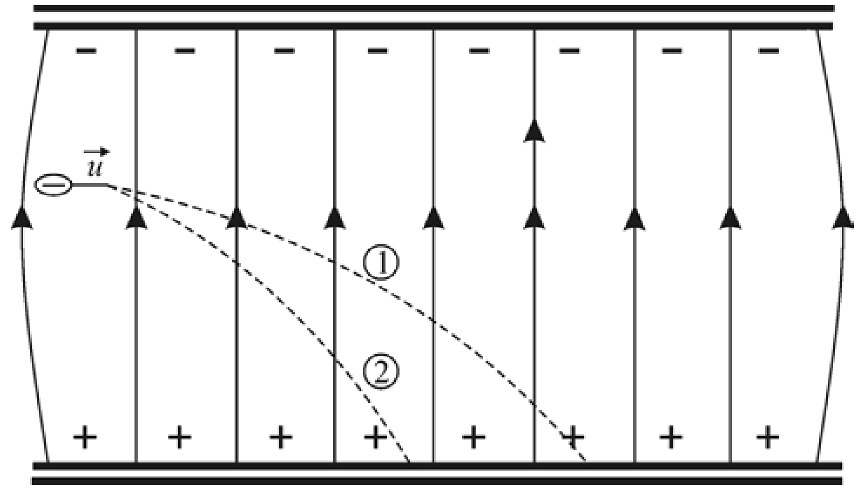


Observable boost

Particle boost

$$\mathbf{E}' = \frac{\mathbf{E}}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad x'(t) = \sqrt{1-\frac{v^2}{c^2}}\,x(t).$$





LORENTZ-VIOLATING ON-MINIMAL COUPLINGS, PAULI EQUATION AND THE AHARONOV-CASHER PHASE

V. A. Kostelecky, S. Samuel; PHYS. REV. D, VOL. 39, N. 2 15 JANUARY 1989

$$\Psi=[\phi(x)+\alpha^{\mu}_{-1}A_{\mu}(x)+\cdots]|0\rangle \ ,$$

S. M. Carroll, G. B. Field, R. Jackiw; PHYS. REV. D, VOL. 41, N. 4 15 FEBRUARY 1990

$$\mathcal{L}_p=\mathcal{L}_{\rm EM}+\mathcal{L}_{\rm CS}$$

$$\mathcal{L}_{\rm CS}=-\tfrac{1}{2}p_\alpha A_\beta \widetilde{F}^{\alpha\beta} \ ,$$

$$\partial_\mu F^{\mu\nu}=4\pi J^\nu+p_\mu \widetilde{F}^{\mu\nu}$$

in terms of components,

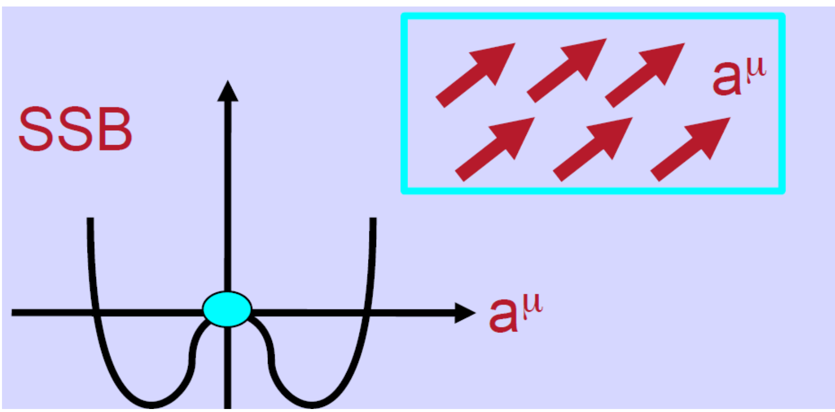
$$\nabla\cdot\mathbf{E}=4\pi\rho-\mathbf{p}\cdot\mathbf{B} \ ,$$

$$-\partial_t\mathbf{E}+\nabla\times\mathbf{B}=4\pi\mathbf{J}-p_0\mathbf{B}+\mathbf{p}\times\mathbf{E} \ .$$

Fermionic Sector

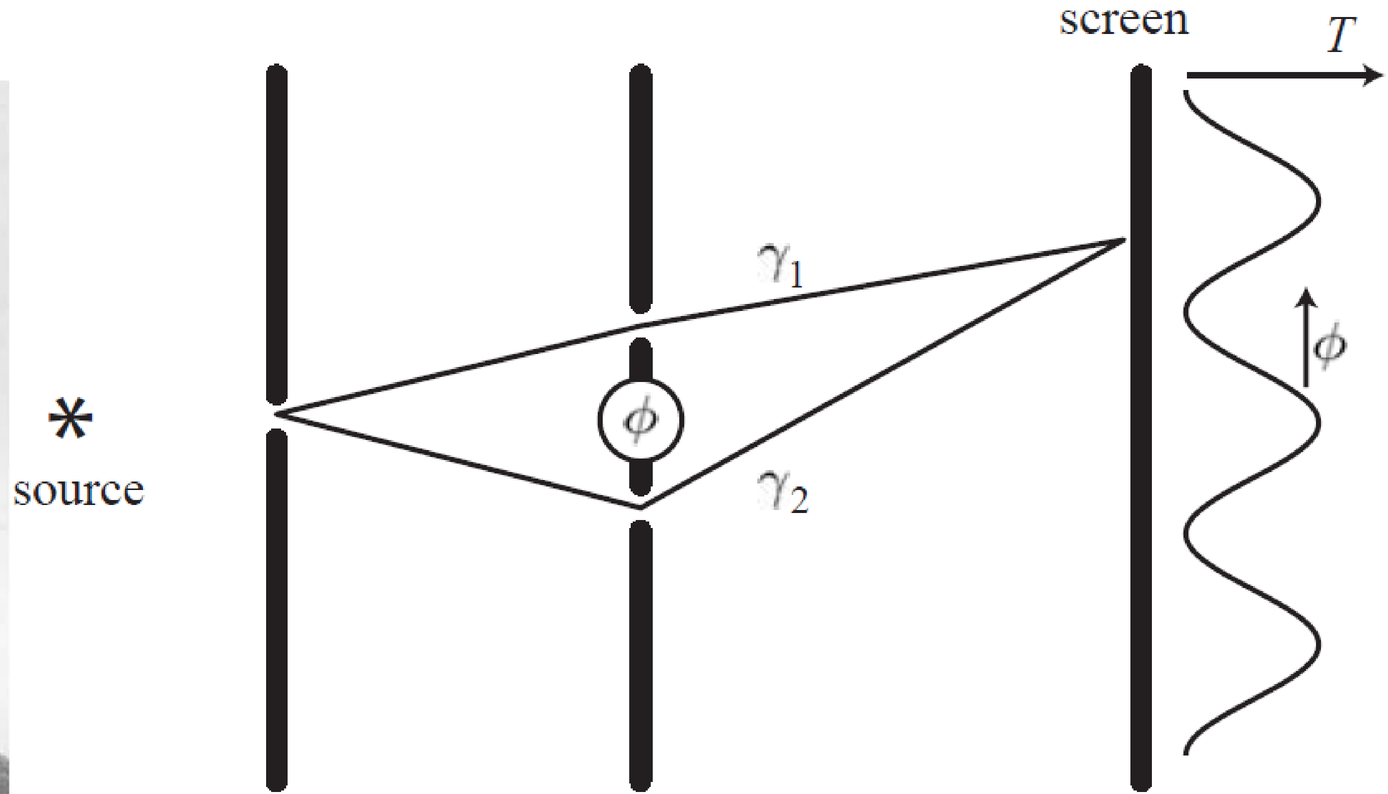
Non-minimal coupling to the gauge field and background

$$L = i\overline{\Psi}\gamma_{\mu}\partial^{\mu}\Psi - m\overline{\Psi}\Psi$$



$$L = i\overline{\Psi}\gamma_{\mu}\partial^{\mu}\Psi - m\overline{\Psi}\Psi + \overline{\Psi}\gamma_{\mu}a^{\mu}\Psi$$

Aharonov–Bohm experiment

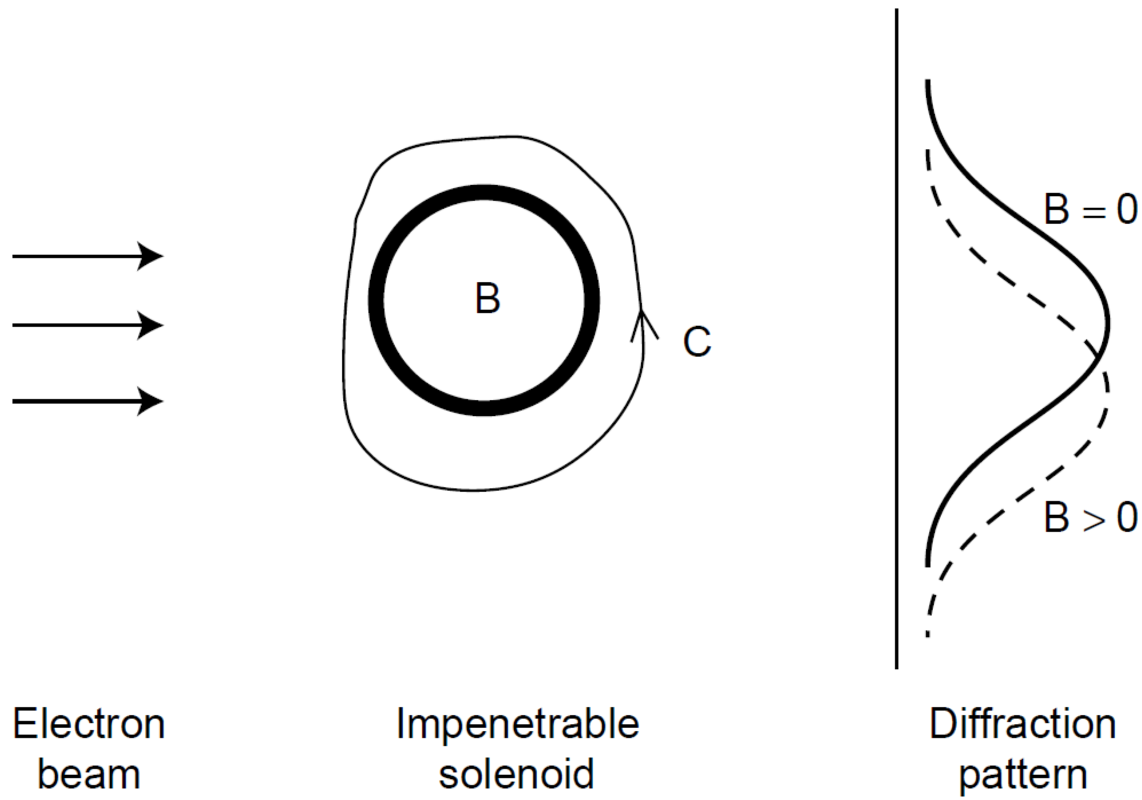


$$\Delta\varphi_{\mathrm{AB}} = -\frac{|e|}{\hbar}\oint \mathbf{A}d\mathbf{s}$$

LORENTZ-VIOLATING ON-MINIMAL COUPLINGS, PAULI EQUATION AND THE AHARONOV-CASHER PHASE

Non-minimal coupling to the gauge field and background

$$\oint \mathbf{A} \cdot d\mathbf{x}$$



The Aharonov–Bohm phase

$$\mathbf{p} \longrightarrow \mathbf{p} + |e|\mathbf{A} \qquad \Delta\varphi_{\mathrm{AB}} = -\frac{|e|}{\hbar}\oint \mathbf{A}d\mathbf{s}$$

Aharonov–Casher phase

$$\mathbf{p} = m\mathbf{v} + \frac{1}{c}\boldsymbol{\mu} \times \mathbf{E}$$

$$\Delta\varphi_{\mathrm{AC}} = \frac{1}{c}\oint \boldsymbol{\mu} \times \mathbf{E}d\mathbf{s}$$

LORENTZ-VIOLATING ON-MINIMAL COUPLINGS, PAULI EQUATION AND THE AHARONOV-CASHER PHASE

Non-minimal coupling to the gauge field and background

$$(i\gamma^{\mu}D_{\mu}-m)\Psi=0,$$

$$D_{\mu}=\partial_{\mu}+eA_{\mu}+igv^{\nu}\widetilde{F}_{\mu\nu},$$

$$\overrightarrow{\Pi}=\left(\overrightarrow{p}-e\overrightarrow{A}+gv^{0}\overrightarrow{B}-g\overrightarrow{v}\times\overrightarrow{E}\right)$$

$$i\gamma^{\mu}\partial_{\mu}\rightarrow i\gamma^{\mu}\partial_{\mu}-g\,b^{\mu}\,F_{\mu\nu}\,(x)\;\gamma^{\nu},$$

$$\left[\vec{p}-g\,b^{0}\,\vec{E}-g\left(\vec{b}\times\vec{B}\right)\right]$$

$$i\gamma^{\mu}\partial_{\mu}\rightarrow i\gamma^{\mu}\partial_{\mu}-\frac{g}{2}\,\eta^{\alpha\beta}\,\bar{F}_{\mu\alpha}\,(x)\;\bar{F}_{\beta\nu}\,(x)\;\gamma^{\mu}\,b_{\lambda}\,\gamma^{\lambda}\,\gamma^{\nu},$$

$$\left[\vec{p}+gb_{0}\left(\vec{E}\times\vec{B}\right)\right]$$

LORENTZ-VIOLATING ON-MINIMAL COUPLINGS, PAULI EQUATION AND THE AHARONOV-CASHER PHASE

Non-minimal coupling to the gauge field and background

$$H = \frac{1}{2m}\vec{\Pi}^2 + e\varphi - \frac{e}{2m}\vec{\sigma}\cdot(\vec{\nabla}\times\vec{A}) + \frac{1}{2m}gv^0\vec{\sigma}\cdot(\vec{\nabla}\times\vec{B}) + \frac{g}{2m}\vec{\sigma}\cdot\vec{\nabla}\times(\vec{v}\times\vec{E})$$

Torsion Non-Minimal Coupling with Lorentz Violation

$$D_\mu = \partial_\mu + eA_\mu + ig_a\gamma_5 v^\nu \widetilde{F}_{\mu\nu},$$

$$\vec{\Pi} = \left(\vec{p} - e\vec{A}\right) \qquad H_{nm} = \vec{\sigma}\cdot\left(g_a v^0\vec{B} - g_a\vec{v}\times\vec{E}\right) + g_a\vec{v}\cdot\vec{B}$$

$$(g\boldsymbol{v}_z) \leq 10^{-14}\ (\mathrm{eV})^{-1}$$

$$\alpha g|\mathbf{v}| < 10^{-8}\ (\mathrm{eV})^{-1}$$

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Obrigado